Background:
The Menger sponge is a fractal shape: a geometric shape which is self-similar in the sense that it can be broken into parts each of which is a reduced-size copy of whole shape. It was first described by Karl Menger in 1926 (Karl Menger was a professor at IIT from 1946 to 1971).

Although it is impossible to build an exact physical representation of a Menger sponge (an object which has infinite surface area and zero volume), we can build approximations.

Here description for constructing approximations of the Menger sponge.

1. Begin with a cube

2. Subdivide this cube into 27 subcubes of equal size.

3. Remove the center subcube on each of the six faces and from the center of the original cube. This will leave you with 20 subcubes and this object is known as a level-1 Menger sponge.

4. Repeat steps 2 and 3 for each of the remaining subcubes to obtain a level-2 Menger cube.

5. Continue repeating steps 2 and 3 to obtain higher order approximations of the Menger sponge.

The limit (or intersection) of the infinite sequence of level-n sponges is, by definition, the Menger sponge.

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Project Exercises:

1. Build a physical model of a level-1 Menger sponge (this can be done with post-it notes).

2. Assuming that one starts with a unit cube (a cube of side length one), determine a formula for the volume and surface area of
   (i) a level-1 Menger sponge
   (ii) a level-2 Menger sponge
   (iii) a level-$n$ Menger sponge for arbitrary $n$

3. Discuss the meaning of the seemingly paradoxical statement that the Menger sponge has infinite surface area and zero volume.

4. Use a computer algebra system (e.g. Mathematica or Matlab) to produce a graphical representation of level-$n$ Menger sponges for (at least) $n = 1, 2, 3, 4$.

5. Use a computer algebra system to show the result of slicing a level-$n$ Menger sponge into two pieces with a plane that is not parallel to one of the faces of the sponge. Allow the equation of the plane to be specified by user input and show the shape that lies on or underneath the plane.