

## Math 100 Project on Matrices (Lubin)

1. Read and "understand" Chapter 5, Matrices, in Cleve Moler's Experiments with MATLAB. Your degree of understanding might depend on your prior knowledge of matrices but everyone should be fluent in the matrix operations and have a basic idea of the concept of a  $2 \times 2$  matrix being a transformation of 2-dimensional vectors, or a mapping from the plane to itself.
2. Do the exercises listed in Moler's Chapter 5.
3. Draw some sketches illustrating some non-linear transformations on the plane. You might look for some examples in your calculus book in the section on changing variables for double integrals. Do you see any matrices lurking in those problems?

4. Consider the matrices

$$P_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}, \quad P_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \quad P_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What do these matrices represent geometrically?

5. Consider  $P_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$ ,  $\sqrt{2}P_y = \begin{pmatrix} 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{pmatrix}$ , and  $P_z = \begin{pmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

and the product  $P = P_x P_y P_z$ .

Discuss the action of  $P$ . Consider and justify the following diagram. ( $\mathbf{v}$  represents the vector

$$\mathbf{v} = \mathbf{i} + \mathbf{j} = (1, 1, 0)^T$$

