

## Math 578 – Computational Mathematics II

**Time and Location:** 1:50--3:05 TR, Location E1 244

**Instructor:** Greg Fasshauer

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**Office hours:** TR: 1:00--2:00, also by appointment

**Textbook(s):** A. Iserles, *A First Course in the Numerical Analysis of Differential Equations*, Cambridge University Press (1996), ISBN 0-521-55655-4 (paperback).

D. Kincaid and W. Cheney, *Numerical Analysis: Mathematics of Scientific Computing*, 3rd Ed, Brooks/Cole (2002), ISBN 0-534-38905-8.

**Other required material:** Matlab

**Prerequisites:** MATH 350 Introduction to Computational Mathematics, or consent of the instructor

### Objectives:

1. Students will understand the basic numerical methods for solving initial value problems and their derivations.
2. Students will understand the concepts of order, stability, and convergence of a numerical method.
3. Students will understand the basic numerical methods for solving boundary value problems and their derivations.
4. Students will learn how to implement and use these numerical methods in Matlab (or another similar software package).
5. Students will improve their problem solving skills in computational mathematics.
6. Students will improve their presentation and writing skills.

### Course Outline:

|   | Hours |
|---|-------|
| 1. Mathematical background  | 10    |
| a. Lipschitz continuity   |       |
| b. Taylor polynomials and polynomial interpolation, splines               |       |
| c. Numerical integration methods  |       |
| d. Richardson Extrapolation   |       |
| e. Existence and uniqueness theorem for initial value problems            |       |
| 2. Nonlinear algebraic systems  | 3     |
| a. Fixed-point iteration  |       |
| b. Newton-Raphson iteration   |       |
| 3. Single step methods for differential equations                         | 5     |
| a. Derivation of Euler and Taylor methods, trapezoidal rule, theta method |       |
| b. Order and convergence  |       |
| 4. Multistep methods for differential equations                           | 7     |
| a. Derivation of Adams methods, general multistep methods, BDFs           |       |

- b. Order and convergence
  - c. Dahlquist equivalence theorem
- 5. Runge-Kutta methods 3
  - a. Derivation
  - b. General form
- 6. Stability and Stiff equations 3
  - a. Linear stability analysis
  - b. Stiffness
  - c. A-Stability
- 7. Error control 4
  - a. Adaptive stepsize control
  - b. Predictor-Corrector methods
  - c. Embedded Runge-Kutta methods
- 8. Boundary value problems 7
  - a. Shooting methods
  - b. Finite differences
  - c. FFT and spectral method

|                    |                                     |     |
|--------------------|-------------------------------------|-----|
| <b>Assessment:</b> | Homework                            | 20% |
|                    | Computer Programs/Project           | 20% |
|                    | Midterm (March 20)                  | 30% |
|                    | Final Exam (May 11, 8:00am-10:00am) | 30% |