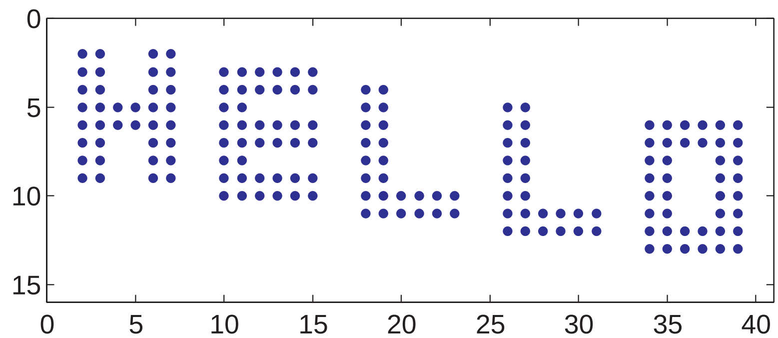
Math 477/577 — Computer Assignment 3, due Oct.12, 2006

1. Write a Matlab function `[Q,R] = mgs(A)` (see the discussion in the classnotes of stability of the Gram-Schmidt algorithms) that computes a reduced QR factorization $A = \hat{Q}\hat{R}$ of an $m \times n$ matrix A with $m \geq n$ using modified Gram-Schmidt orthogonalization. The output variables are a matrix $Q \in \mathbb{C}^{m \times n}$ with orthonormal columns and a triangular matrix $R \in \mathbb{C}^{n \times n}$.

2. (a) Write a Matlab program that sets up a 15×40 matrix with entries 0 everywhere except for the values 1 in the positions indicated in the picture below. The upper-leftmost 1 is in position (2,2), and the lower-rightmost 1 is in position (13,39). This picture was produced with the command `spy(A)`.



   (b) Call `svd` to compute the singular values of A, and print the results. Plot these numbers using both `plot` and `semilogy`. What is the mathematically exact rank of A? How does this show up in the computed singular values?

   (c) For each i from 1 to rank(A), construct the rank-i matrix B that is the best approximation to A in the 2-norm. Use the command `pcolor(B)` with `colormap(gray)` to create images of these various approximations.

3. (a) Write a Matlab function `[W,R] = house(A)` that computes an implicit representation of a full QR factorization $A = QR$ of an $m \times n$ matrix A with $m \geq n$ using Householder reflections. The output variables are a lower-triangular matrix $W \in \mathbb{C}^{m \times n}$ whose columns are the vectors $\boldsymbol{v}_k$ defining the successive Householder reflections, and a triangular matrix $R \in \mathbb{C}^{n \times n}$.

   (b) Write a Matlab function `Q = formQ(W)` that takes the matrix W produced by `house` as input and generates a corresponding $m \times m$ orthogonal matrix Q.

4. Let Z be the matrix

$$Z = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 7 \\ 4 & 2 & 3 \\ 4 & 2 & 2 \end{bmatrix}.$$

   Compute the reduced QR factorization of Z in Matlab: by the Gram-Schmidt routine `mgs` of Problem 1, by the Householder routines `house` and `formQ` of the previous problem, and by Matlab's built-in command `[Q,R] = qr(Z,0)`. Compare these three and comment on any differences you see.