Math 100 Project: Estimate Travel Times on Freeways

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1 Introduction

For metrapolitan areas, traffic reports are usually provided on radios, TVs, mobile devices and internet. An example is the web site http://travelmidwest.com. Most traffic reports give estimated travel times on various freeways. In this project, we try to find out how these estimations are created. We will learn what data are collected for the estimation, how to set up a mathematical model, numerical integration and interpolation techniques.

2 List of Tasks

1. (Collect information)

   On the internet, try to find out how the traffic report on travel times on freeways are created, such as travelmidwest.com and traffic.com.

2. (Set up mathematical model)

   Suppose a segment of a freeway goes from $x = 0$ to $x = L$, where L is the total distance of this segment. There are car speed sensors that record the current car speed at n different locations $\{x_i, i = 1, 2, \cdots, n\}$, where $0 = x_1 < x_2 < \cdots < x_n = L$. Find a formula for computing the total travel time of a car from point $x = 0$ to $x = L$, if we knew the car speed $v(x)$ at any point on the interval $[0, L]$.

   (a) Background: Speed or velocity v of a car is dx/dt, where t is the time variable. v is provided as a function of location x. Let's denote the total travel time spent by a car going from location

$x = 0$ to $x = L$ by T. The travel time T can be written as integrals

$$T = \int_0^T \mathrm{d}t = \int_0^L \frac{\mathrm{d}t}{\mathrm{d}x}\,\mathrm{d}x, \tag{1}$$

where $t = t(x)$ is the time when the car is at the location x on the freeway.

Justify the equality in (1) using one of the integration techniques in calculus.

(b) Figure out what is the relation between this function $t = t(x)$ and the speed of the car $v = v(x)$. Hint: Think about inverse functions and derivatives of inverse functions in calculus.

3. (Numerical Integration)

Review calculus on the concepts of Riemann sums for defining integrals. Learn the numerical integration techniques such as the mid-point rule, the trapezoidal rule, and the composite trapezoidal rule. One resouce on numerical integration can be found in §8.7 of the Calculus textbook by Stewart (6th ed).

4. (Estimation)

(a) Using numerical integration technique of your choice, estimate the total travel time T from the following hypothetical data. There are speed sensors located at $x = 0, 2, 4.5, 8, 11.2, 15.4, 18.6, 20.8, 24.1, 27.5, 30$ (miles), and the readings of car speed v at these locations are $52, 40, 19, 9, 22, 34, 54, 41, 15, 37, 54$ (miles per hour) respectively.

(b) One can find actual speed data at the site travelmidwest.com. For example,
http://www.travelmidwest.com/lmiga/detectors.jsp?location=GATEWAY.IL.I-55.

Using numerical integration, estimate the travel time using your scheme and compare with the travel times displayed on the web site. Note that you need to find the distances between the different locations on the freeway, using the internet such as Google maps.