Math 478/578: Computer Assignment 1 — due Tuesday, Feb. 13, 2007

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1. Consider the following system of second-order initial-value problems:

$$x''(t) = -\frac{x(t)}{(x(t)^2 + y(t)^2)^{3/2}}, \qquad x(0) = 1,\ x'(0) = 0, \tag{1}$$

$$y''(t) = -\frac{y(t)}{(x(t)^2 + y(t)^2)^{3/2}}, \qquad y(0) = 0,\ y'(0) = 1. \tag{2}$$

These are Newton's equations of motion for the two-body problem. Here the pair $(x(t), y(t))$ describes the trajectory of one of the bodies at time t. If we let t range from 0 to 2π, then the solution will be a circle.

(a) Transform the given problem into an appropriate system of first-order initial-value problems.

(b) Write a Matlab function `Twobody.m` that calculates the (vectorized) right-hand side of the system obtained in the previous step. The function should be of the form

```
function yprime = Twobody(t, y),
```

where `yprime` and `y` are appropriate column vectors.

(c) Write a Matlab driver script that solves the equations of motion with Matlab's built-in ODE solvers `ode23` and `ode45`. The calling sequence for both of these functions is of the form

```
[t, y] = ode23(f, [tstart tend], y0),
```

where `f` is the name of the right-hand side function (your function `Twobody`), `tstart` and `tend` are starting and ending t-values, and `y0` is a (column vector) of initial conditions.

(d) Plot the computed solutions in the xy-plane. For debugging/testing purposes you should also include plots of the components of the vector `y` against t. This can be accomplished in a single plot by using Matlab syntax such as

```
subplot(2, 2, 1), plot(t, y(:, 1))
subplot(2, 2, 2), plot(t, y(:, 2))
subplot(2, 2, 3), plot(t, y(:, 3))
subplot(2, 2, 4), plot(t, y(:, 4))
```

(e) Modify the Matlab functions `Euler.m` and `Trapezoid.m` presented in class so that they work for systems of first-order initial-value problems.

(f) Solve the system (1), (2) again via Euler's method and the trapezoidal rule and compare your answers to Matlab's solutions obtained in (c) and (d). Comment.